INDIVIDUAL VARIATIONS IN ARITHMETIC

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One of the most significant features of the various studies that have been made in arithmetic is the wide variation in the achievements of the pupils of any grade. The purpose of this study is to determine some of the causes of individual variations in arithmetic and to note the effects of certain corrective exercises

With this end in view, the arithmetic tests used in the Cleveland Survey[1] were given to Grades IIIB–VIIB of the Elementary School of the University of Chicago and the results were used to diagnose the points of individual weaknesses.

The complete set of results of these tests cannot be included in this article. These results showed a large amount of variation in both individuals and classes. There were pupils in the fourth grade who accomplished more in the same time than some pupils in the seventh grade. Furthermore, the best pupil in each grade was approximately three times as rapid as the poorest. Some of the pupils were very good in part of the fifteen sets of the test and extremely poor in other sets. This wide variability suggested the necessity of a more detailed study of certain individuals with particular emphasis on their points of weakness as shown by the results of the test.

Individual tests were then prepared and given to 88 pupils in Grades VA–VIIB. These tests were composed of examples from the regular test, but they were adapted to each individual in order to determine the nature of his difficulties in the sets in which he made low scores. Each pupil was asked to work the examples orally so that any mistakes could be definitely located. Notes were kept on a note pad and later copied on the individual's test

[1] C. H. Judd, *Measuring the Work of the Public Schools*. Published by the Survey Committee of the Cleveland Foundation, 1916.

sheet. The following reproduction shows a portion of one of the individual tests:

Subtract:

9	7	11	8	12	1	9	13	4	12
9	3	6	1	3	0	7	8	3	6
0	5	4	7	9	1	3	5	1	6

Time 32 sec.

Subtract:

1335	816	1157	854	said 8 from 15
419	335	908	286	"borrowing".
927	481	249	578	Time 57 sec.

Divide:

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      571
69 ) 40296
     335     mult. error.
     579
     483     Subt. errors
      85
      69
      17
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Time 2 min. 37 sec.
Failed to notice that the remainder 85 was larger than the divisor.
Failed to notice neglect of carrying down the 6.

This individual test showed the pupil to be very weak in subtraction. Note that he missed 3 out of the 10 easy subtractions at the beginning of the test. The example in long division showed an incomplete understanding of that process, but even if he had understood the division process, he would have been unable to work the example correctly on account of making so many errors in the subordinate processes of multiplication and subtraction.

A careful analysis of the difficulties of each pupil was made and the errors of the 88 pupils tested were tabulated under the following headings:

Nature of the Error	No. of Errors
Incorrect addition, subtraction, and multiplication responses made in working long problems in multiplication and division	39
Incorrect addition responses in addition examples	14
Incorrect subtraction responses in subtraction examples	7
Incorrect multiplication responses in multiplication examples	7
Incorrect division responses in division examples	13
Carrying and copying figures	35
One process, through suggestion, carrying over into the next process	14
Trouble with the symbolism, particularly in fractions	25
Lack of knowledge of the process	28

This table shows some of the main causes of failure on the part of the pupils. One of the chief difficulties is the inability of the pupil to carry on two or more processes simultaneously. A large number of errors is due to the incomplete automatization of the simple facts of addition, subtraction, multiplication, and division. This defect comes out particularly when the attention is centered upon another process, as shown in the first item in the foregoing table. Psychological experiments have shown that two complex mental processes cannot be carried on at the same time without interference, unless one is made automatic. This study indicates then that more drill should be given to the tables of the four fundamental operations if we expect them to function properly in complex situations. The large number of errors in "carrying" and copying figures is also probably due to the attention being focused upon another part of the process rather than to the usual explanation of "carelessness."

Failures in fractions were due in many cases to lack of knowledge of the process. In other cases pupils failed to note the symbolism and could give the correct solution when the examples were read to them. Suggestion also played an important rôle in fractions. Many of the pupils carry the process of addition of fractions over into the solution of problems calling for subtraction and multiplication. They were very much surprised at the close of the test when their attention was called to the subtraction and multiplication signs.

While there are certain general tendencies that can be determined in such a study of individual difficulties, the principal benefit comes from the location by the teacher of points of weakness in each of his pupils. Any teacher will find such a study very useful in his efforts to improve the efficiency of his class.

In order to increase the efficiency of the pupils in the mechanical phases of arithmetic and to correct some of the errors that were observed, three types of drill were attempted: (1) class drill supplemented by individual assistance on the points of weakness as diagnosed by the results of the test; (2) class drill with extra drill periods provided for the slow pupils, who were drilled in groups rather than individually; and (3) merely class drill with explanations to the class as a whole.

The first type of drill was used in Grade VA. The teacher used three periods (of 25 minutes each) per week for four weeks for regular class drill. This class drill was supplemented at other periods in the day by special help to different pupils in the processes in which they were weak, and they were required to work extra examples in those processes after the help had been given. The drill was limited to the four fundamental operations and fractions. At the end of this month of drill, the pupils were given the original test a second time.

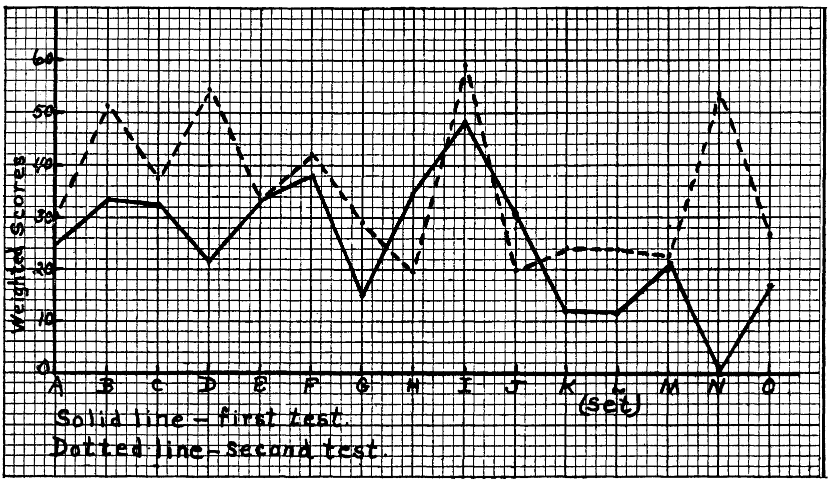

CHART I

Chart I gives an interesting comparison of the results of one of the pupils in Grade VA. The solid line represents the curve for the first test and the dotted line that of the second test. The results of sets B to O inclusive were weighted in such a way that the graph would be a horizontal line if the pupil were equally proficient in all of the fifteen sets of the test. Since the same weights were used in connection with both sets of results it will not be necessary in this comparative study to enter into a detailed description of the derivation of this system of weights.[1] The solid line

[1] The system of weights was derived from the results from all the grades. The method is too involved to be explained in brief form. See Appendix, p. 254, of *Measuring the Work of Public The Schools,* Cleveland Survey, for a table showing the relative difficulty of the fifteen sets of the test. A similar table forms the basis of the system of weights used in this study.

in Chart I showed the pupil to be weak in division (sets D, K, and N). Corrective work was accordingly given on those sets. Note the effect of this assistance in the pronounced upward bends in the curve at sets D and N.

Chart II shows the curves of another pupil of the same grade, who made a very poor record on the first test. He showed exceptional weakness in sets A, B, F, I, N, and O. On the basis of this diagnosis, corrective work was applied in similar examples. His curve for the second test shows a phenomenal gain over that of the

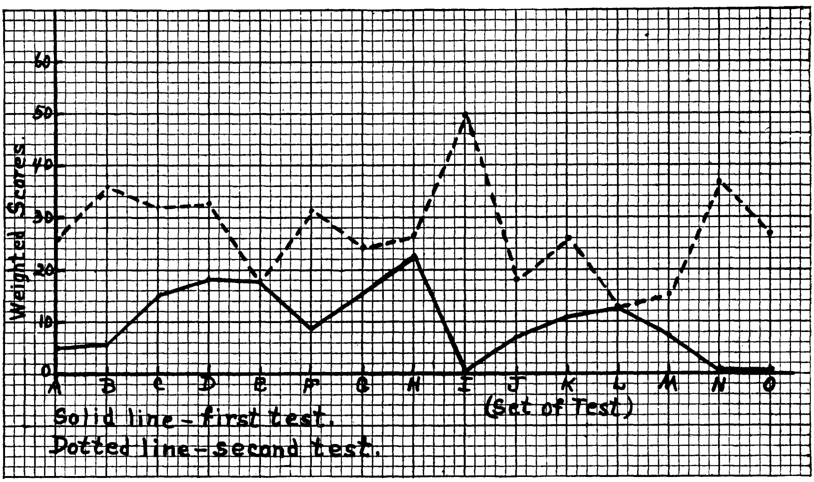

CHART II

first test, but exceptional gains are shown in sets A, B, F, I, N, and O where the corrective work was attempted.

The two charts indicate the excellent results which can be secured by a teacher who uses the results of a standard test to locate points of weakness in his pupils and who applies proper corrective treatment.

In this grade, using the first type of drill, there was an increase in the median total score[1] of the class of 94 per cent and a decrease from 0.53 to 0.21 in the quartile coefficient of deviation[2] in the total scores.

[1] The total score for each pupil is the sum of fifteen weighted scores of the test.

[2] The quartile coefficient of deviation is found by arranging the scores in order from the lowest to the highest. The first quartile is located on this scale one-fourth

In the second type of drill used in Grade VIIB, in which class drill was supplemented with group drills to the slow members of the class, there was an increase of 78 per cent in the median total score of the class and a decrease from 0.47 to 0.35 in the quartile coefficient of deviation.

In Grade VIA, using the third type of drill, class drill supplemented by explanations to the class as a whole, there was an increase of 73 per cent in the median total score and a decrease from 0.41 to only 0.40 in the quartile coefficient of deviation.

A comparison of the results of the three types of drill work shows: (1) All three types of drill produced very large increases in the achievements of the pupils. (2) Class drill supplemented by individual help at the points of weakness as diagnosed by the first test proved much more efficient on account of the exceptional decrease in the variation among the members of the class. This decrease in variation was shown by the decrease in the quartile coefficient of deviation. (3) It has been shown in both the first and second types of drill that individual variations which some writers ascribe to hereditary influences may be greatly modified by appropriate instruction.

of the number of scores up the scale. The median is a point on the scale in which half of the scores are above this point and half below. The third quartile is located three-fourths of the distance up this scale. The quartile coefficient of deviation is found by subtracting the first quartile from the third quartile and dividing this difference by the median score. The wider the variation in a class's scores, the larger will be the quartile coefficient of deviation, and vice versa.